Defining the Global Error of a Multi-Axis Vibration Test: An Application of MIL-STD-810G Method 527 Annex C

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MIMO Concepts & Testing, Part 1

Importance of Multi-Axis Vibration Testing

- Multi-axis vibration testing is used to accurately replicate dynamic environments
  - Sometimes referred to as Multiple-Input Multiple-Output (MIMO) testing
- Multiple mechanical degrees of freedom (DOF) can be excited simultaneously
  - Can be a combination of translational and/or rotational DOF
- Provides a more accurate real-world test environment over single axis testing
  - Multi-axial field measurements can be reproduced in the test lab
- Applications
  - Multi-axial fatigue studies
  - Multi-axial modal analysis & model validation
  - Accelerated Life-cycle testing
  - Real world product screening

Control of MIMO Vibration Tests

- Minimal control requires one measurement per excitation DOF
  - Measurements must be oriented properly to resolve required DOFs
  - e.g. – 6 DOF testing requires a minimum of 6 control accelerometers capable of measuring all DOF
- Vibration control algorithms allow for multiple measurements per DOF
  - Over-determined feedback (rectangular) control – more measurements than control points (MIMO Averaging)
  - Common for a 3-DOF test to use 12 measurement accelerometers
  - MIMO testing requires significantly more measurements than single axis
- Existing single axis standards can be applied as a simultaneous multi-axis test
  - Given no CSD terms, statistical independence between mechanical DOF's is generally assumed
  - Special consideration may be required for payloads with closely coupled modes

Control 6-DOF

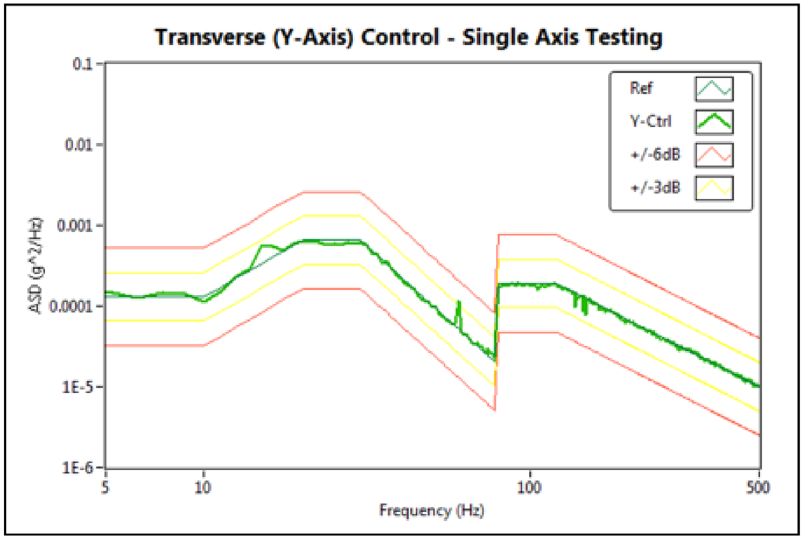
$$\begin{Bmatrix} X \\ Y \\ Z \\ R_x \\ R_y \\ R_z \end{Bmatrix} = \begin{bmatrix} 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 & 0 & 0 & 0.25 \\ 0 & 0 & 1.23 & 0 & 0 & 1.23 & 0 & 0 & -1.23 & 0 & 0 & -1.23 \\ 0 & 0 & -1.23 & 0 & 0 & 1.23 & 0 & 0 & 1.23 & 0 & 0 & -1.23 \\ -0.62 & 0.62 & 0 & -0.62 & -0.62 & 0 & 0.62 & -0.62 & 0 & 0.62 & 0.62 & 0 \end{bmatrix} * \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ x_3 \\ y_3 \\ z_3 \\ x_4 \\ y_4 \\ z_4 \end{Bmatrix}$$

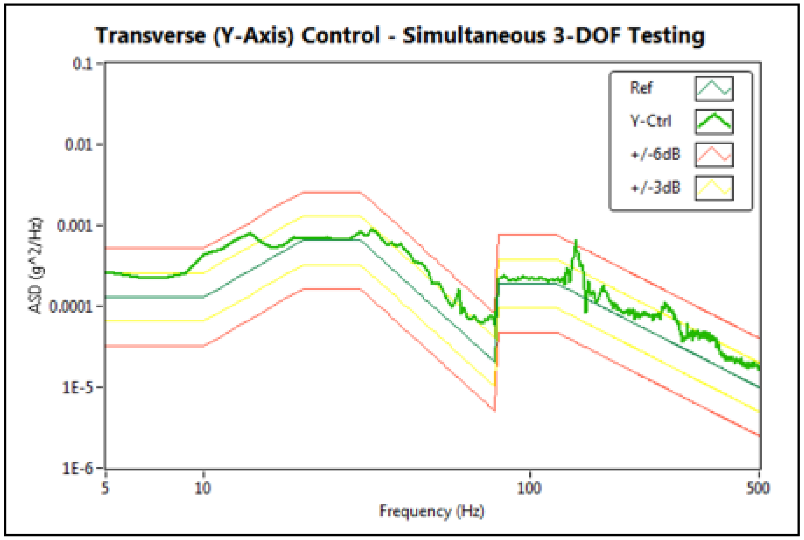
Input Signal Transformation Matrix

Measurement Accelerometers

MIMO Considerations

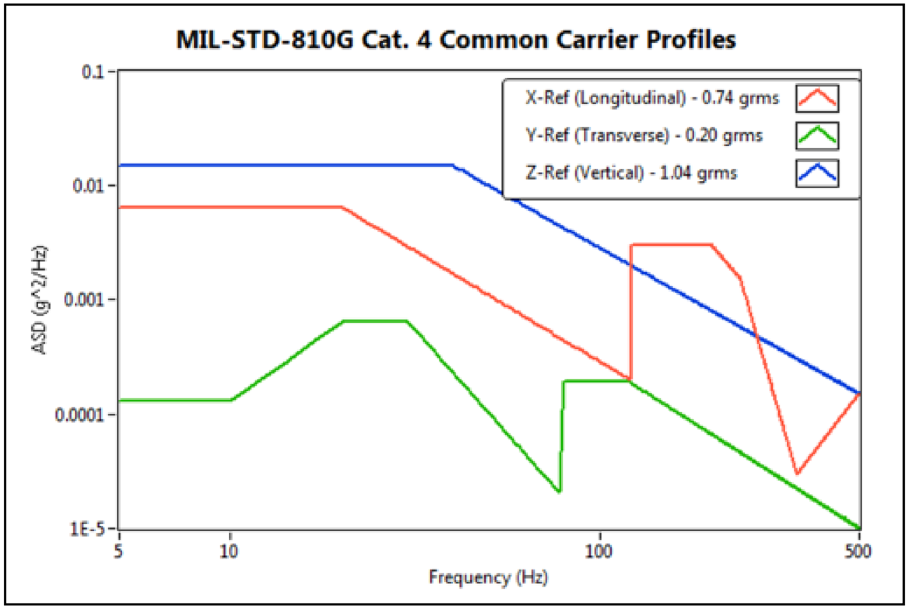
- Control of the lowest level profile is often the most difficult
- Accuracy of low level profile may degrade with 3-axis simultaneous excitation vs. single axis
  - e.g. - MIL-STD-810G Common Carrier Test – Transverse profile
- Possible Sources:
  - Cross-coupling between axes
  - Approaching noise floor of the multi-axis system
  - Exciting non-linear harmonics of the test article
- Error associated with low-level profile may dominate the control
- Is this acceptable or even logical?





Reasons for Quantifying a Global Error

- Excitation DOFs have significantly different Auto Spectral Density (ASD) levels
  - An order of magnitude or greater difference in MIL-STD-810G Cat. 4 Common Carrier profiles
- Traditional single DOF tolerances can force unreasonable constraints for MDOF testing
  - +3dB relative error on a DOF **one** order of magnitude lower than the highest profile (Z) is a change from 10% to 20% of Z
  - +3dB relative error on a DOF **two** orders of magnitude lower than the highest profile (Z) is a change from 1% to 2% of Z
  - Stringent guided single axis tests allow 20% cross-axis motion
- Cross-axis motion from high-level profiles may be at the same level as the low profile
- Are single axis tolerances/abort levels applicable to the MIMO case?
- Should there be different allowable errors for different levels within a MIMO test?



MIL-STD-810G Method 527 Annex C Global Error

- Method 527, Annex C discusses the concept of using a Global Error metric
- Logic is rooted in MIMO control difficulties with different ASD levels
  - Hale examined this concept – 77th Shock and Vibration Symposium, 2006
- Method 527 develops a weighting function algorithm that accounts for differences in DOF levels
- Places greater emphasis on the error of the high energy DOF
- Annex C describes an algorithm for both Time-Waveform-Replication and Auto Spectral Density random tests.

MIL-STD-810G
METHOD 527 ANNEX C

7. A Global ASD error may now be established for each time segment s as illustrated in Equation 3.3.7.

$$\underset{1xF}{Glob\_asd\_err_s} = \left(\sum_{j=1}^{J} (ASD\_Nerr) U_1, \cdots, \sum_{j=1}^{J} (ASD\_Nerr) U_F \right) \qquad (3.3.7)$$

The ASD error spectrum produced in Step 7 above provides a global perspective to ASD error between the reference and laboratory data in which each control location is included, and weighted in terms of the energy at each spectral line.

ASD Global Error Algorithm

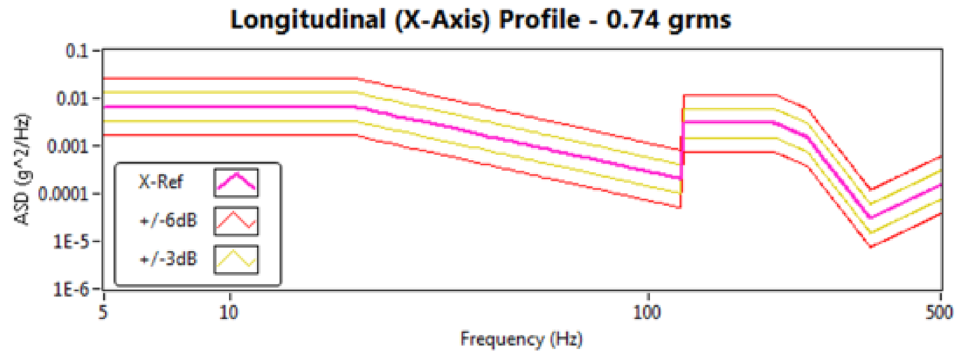
1. Measure Auto Spectral Density of control channels in a vibration test
2. Compile the ASD of the measured control channels & reference profiles into separate matrices
3. Compute a Normalizing factor of the reference profile at each frequency line
4. Combine the reference profiles and the normalizing factor into a Weighting factor matrix
5. Calculate the Relative error of each measurement channel to its reference profile
6. Normalize the relative error matrix with the weighting matrix
7. Sum the normalized error matrix components at each frequency line for a Global Error spectrum

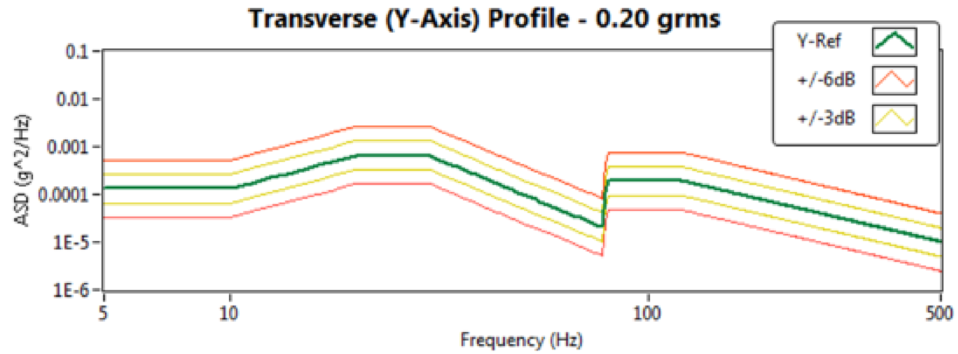
Applying the Theory: Bridged Dual CUBE System

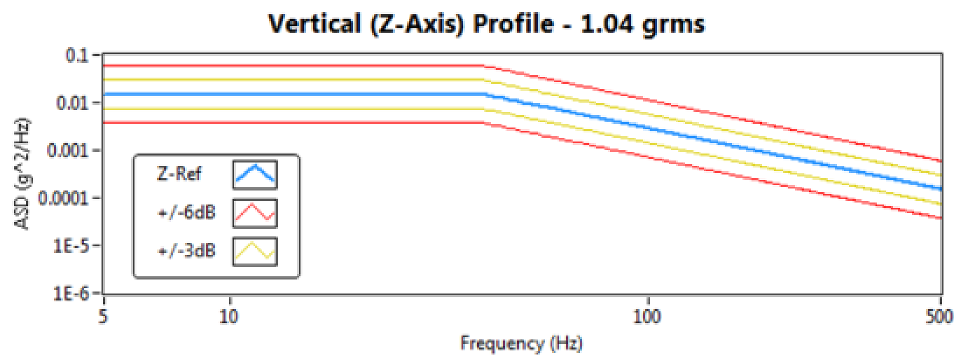
- Dual Cubes with a bridging test article
  - 3,630 lbm test article
  - 163″ L x 40″ W x 31″ H
  - Distributed evenly between Cube
- Contains an inner isolated mass
  - ~9 Hz rigid body mode
- Dual Cubes operated as a single 6-DOF system
- Complex Control Scheme
  - 6 excitation points per Cube (12 total)
  - 4 tri-axial accelerometers per Cube (8 total)
  - 12 measurement control channels per Cube (24 total)
  - Excite 3 translation DOF
  - Minimize 3 rotational DOF
  - Rectangular control

Control Requirements

- Customer
  - Naval Packaging, Handling, Storage, and Transportation (PHST) Center NSWC IHEODTD – Picatinny Detachment
- Requirements
  - Perform MIL-STD-810G Common Carrier Transportation Profiles in X, Y and Z both sequentially and simultaneously
  - 5-500 Hz Bandwidth







Steps 1 & 2: Compile Measurement & Reference Profiles

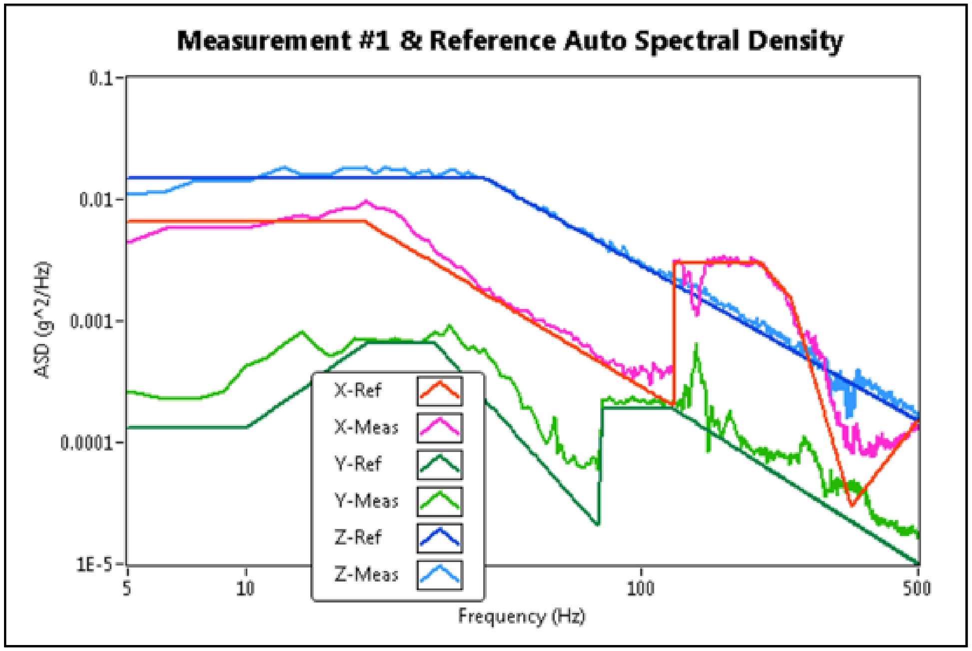
- Assemble the measurement channels and reference profiles into ASD matrices
- Matrix Rows: Number of measurements at each control point or total number of measurement points (j)
  - Data presented compares the X, Y, Z response at each point – (3) measurement rows
- Matrix Columns: Number of frequency lines in measurement ASD (f)

Reference ASD Matrix

$$\boldsymbol{R_{jxf}} = \begin{bmatrix} r_{11} & \cdots & r_{1f} \\ \vdots & \ddots & \vdots \\ r_{j1} & \cdots & r_{jf} \end{bmatrix}$$

Measurement (Lab) ASD Matrix

$$\boldsymbol{L_{jxf}} = \begin{bmatrix} l_{11} & \cdots & l_{1f} \\ \vdots & \ddots & \vdots \\ l_{j1} & \cdots & l_{jf} \end{bmatrix}$$



Steps 3 & 4: Normalizing Factor & Weighting Matrix

- Normalize the reference profiles at each frequency line
  - Provides insight to the relative levels of the reference DOFs
  - L2 Norm is a common normalizing method – essentially an RMS
- Calculate the Weighting matrix
  - Ratio of the squares of the reference and L2 norm
  - Spectra highlight the dominant axes vs. frequency

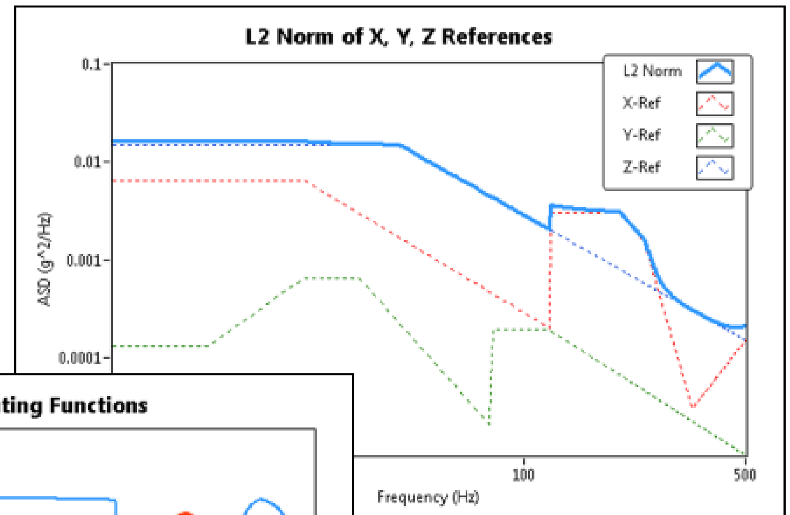
L2 Normalizing Function

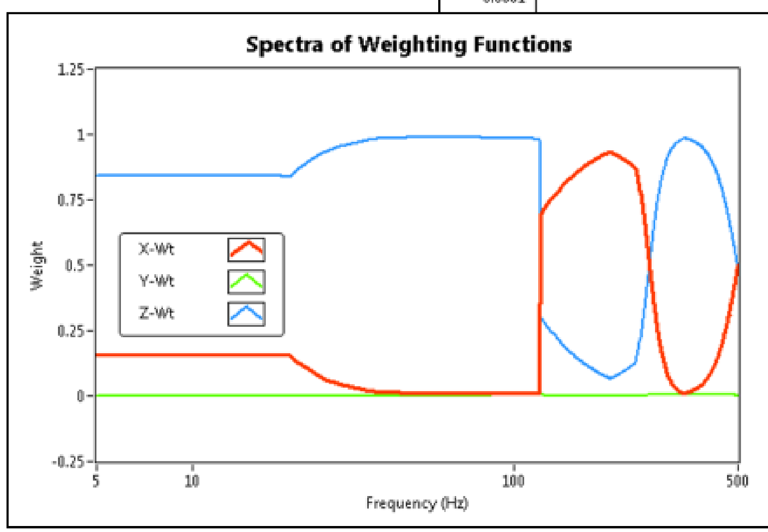
$$\boldsymbol{\eta}_{\boldsymbol{1xf}} = \left[\|R_1\| \quad \|R_2\| \quad \cdots \quad \|R_f\| \right]$$

$$\|R_1\| = \sqrt{r_{11}^2 + r_{12}^2 + \cdots + r_{1j}^2}$$

Weighting Matrix

$$\boldsymbol{W}_{\boldsymbol{jxf}} = \begin{bmatrix} \frac{r_{11}^2}{\eta_1^2} & \cdots & \frac{r_{1f}^2}{\eta_f^2} \\ \vdots & \ddots & \vdots \\ \frac{r_{j1}^2}{\eta_1^2} & \cdots & \frac{r_{jf}^2}{\eta_f^2} \end{bmatrix}$$





Steps 5 & 6: Relative & Normalized Error

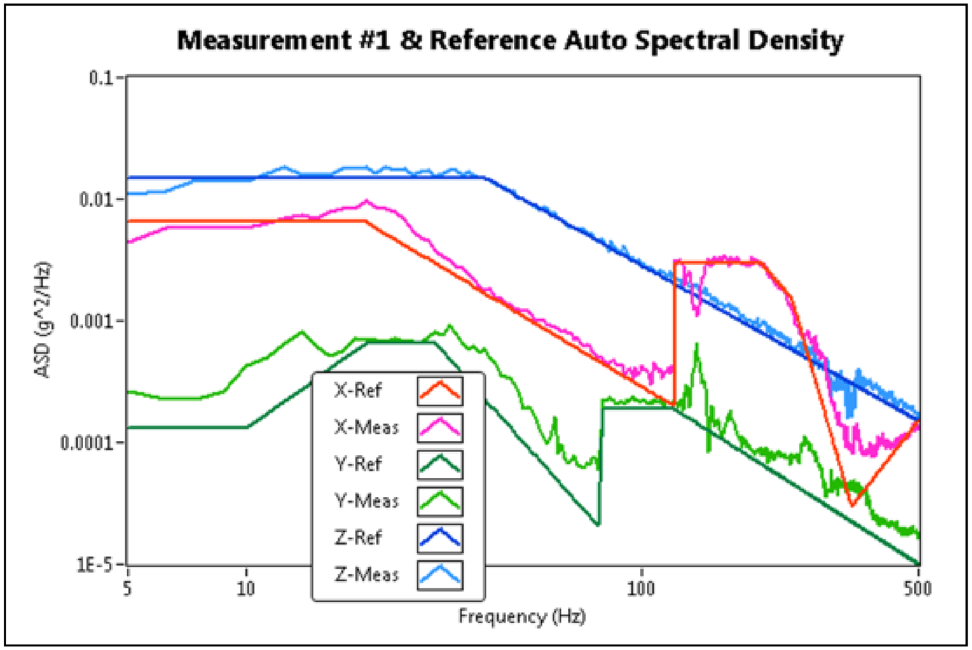
- Calculate relative error
  - Ratio of the measurement to its reference at each frequency line for each measurement DOF
  - Typical tolerance is ±3dB for a single axis vibration test
- Normalize the relative error
  - Apply the Weighting matrix to Relative error at each frequency line

Relative Error

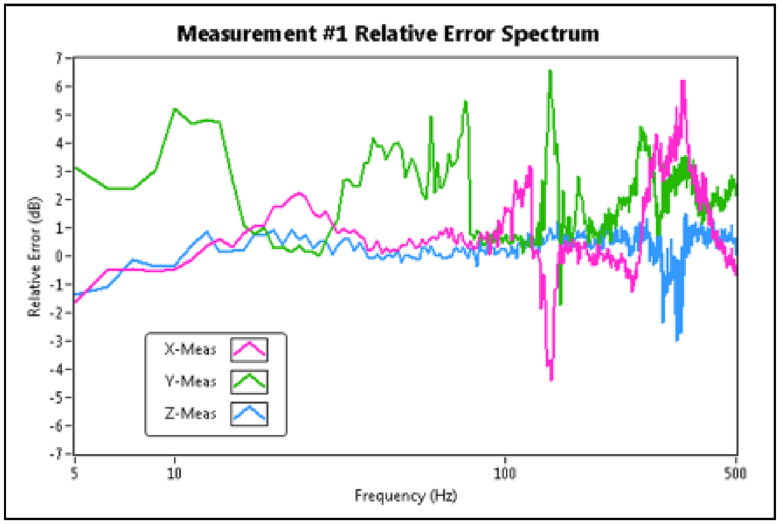
$$\boldsymbol{E_{jxf}} = 10\log_{10}\begin{bmatrix} \frac{l_{11}}{r_{11}} & \cdots & \frac{l_{1f}}{r_{1f}} \\ \vdots & \ddots & \vdots \\ \frac{l_{j1}}{r_{j1}} & \cdots & \frac{l_{jf}}{r_{jf}} \end{bmatrix}$$

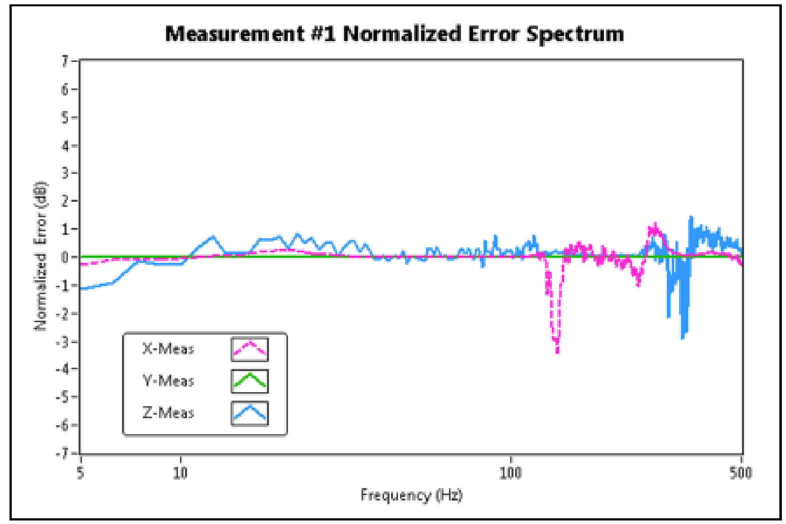
Normalized Error

$$\boldsymbol{N_{jxf}} = \begin{bmatrix} E_{11}\cdot W_{11} & \cdots & E_{1f}\cdot W_{1f} \\ \vdots & \ddots & \vdots \\ E_{j1}\cdot W_{j1} & \cdots & E_{jf}\cdot W_{jf} \end{bmatrix}$$

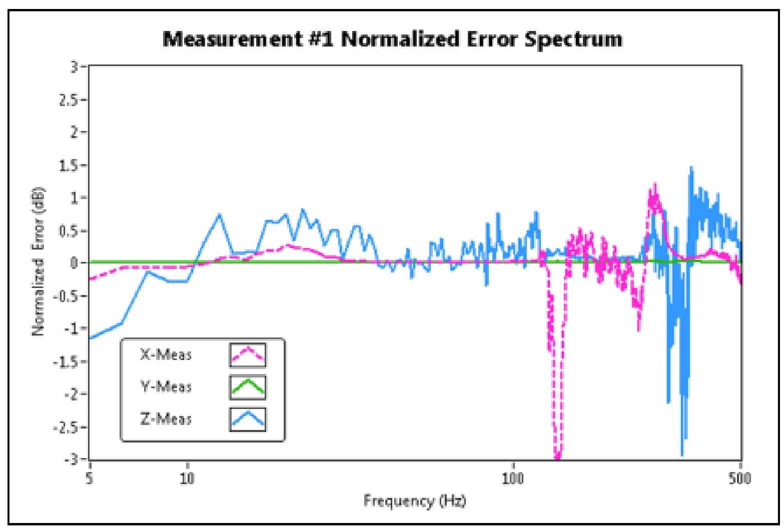


Steps 5 & 6: Relative & Normalized Error (cont.)





Zoomed In



- Y relative error reaches +6dB
  - Dominates relative error plot
- Y normalized error is null
- X & Z normalized errors track relative error
  - Except where significant level differences exist

Step 7: Global Error

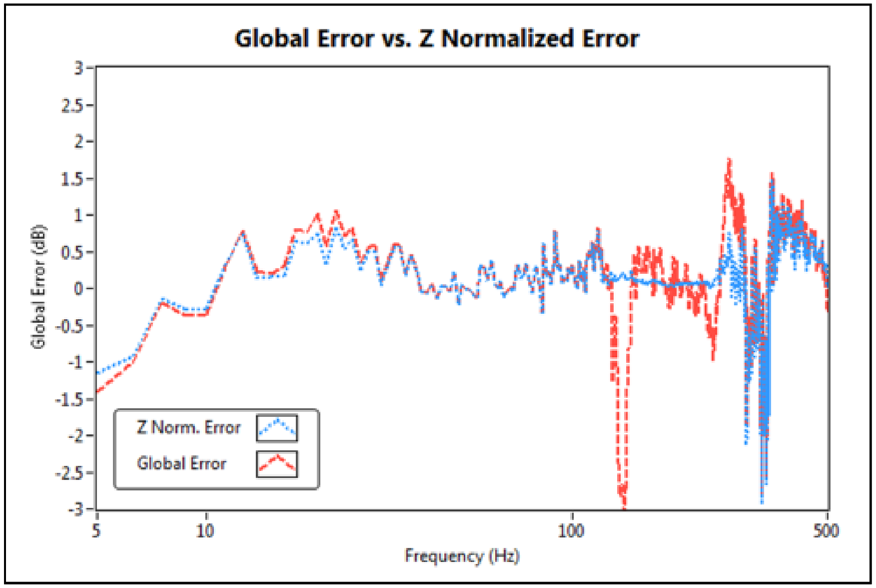
- Sum the Normalized error rows at each frequency line for a single spectrum
- Global Error tracks the highest level profile if there is significant level differences
- Provides a quantitative measure of the overall error of a vibration test

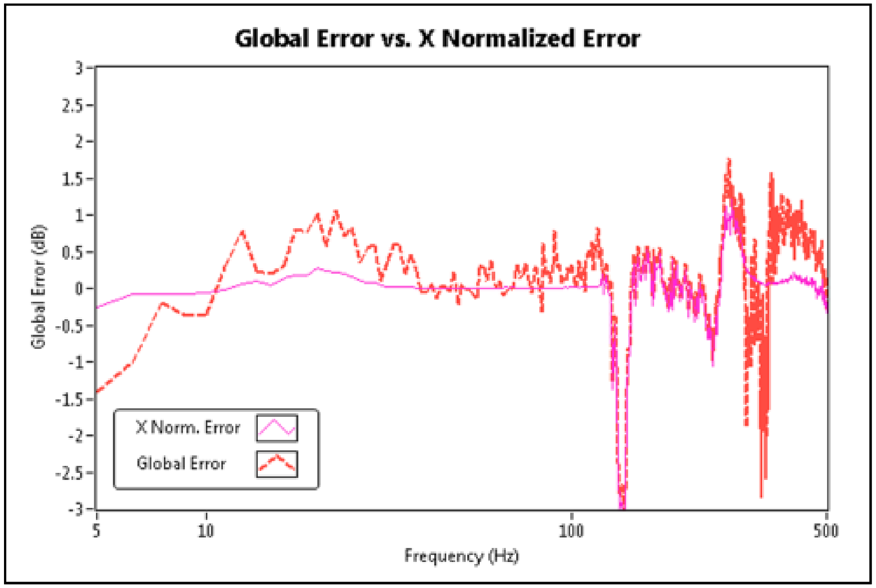
Normalized Error

$$N_{jxf} = \begin{bmatrix} E_{11} \cdot W_{11} & \cdots & E_{1f} \cdot W_{1f} \\ \vdots & \ddots & \vdots \\ E_{j1} \cdot W_{j1} & \cdots & E_{jf} \cdot W_{jf} \end{bmatrix}$$

Global Error

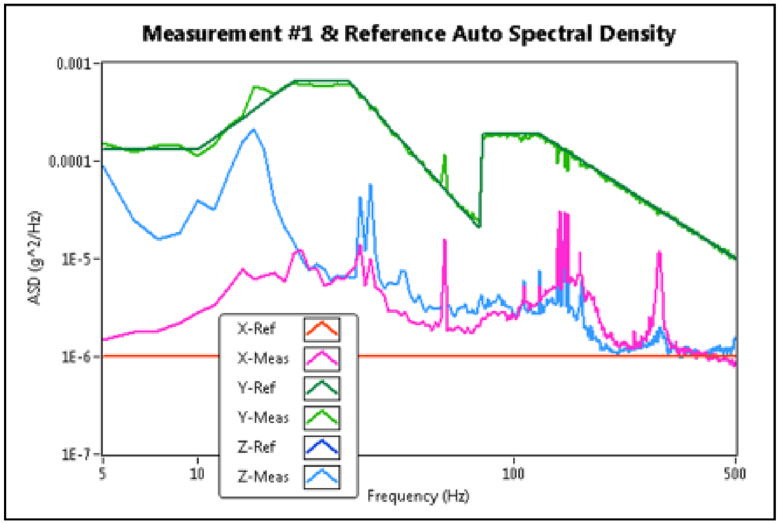
$$\boldsymbol{G_{1xf}} = \begin{bmatrix} \sum_{i=1}^{j} N_{i1} & \cdots & \sum_{i=1}^{j} N_{if} \end{bmatrix}$$

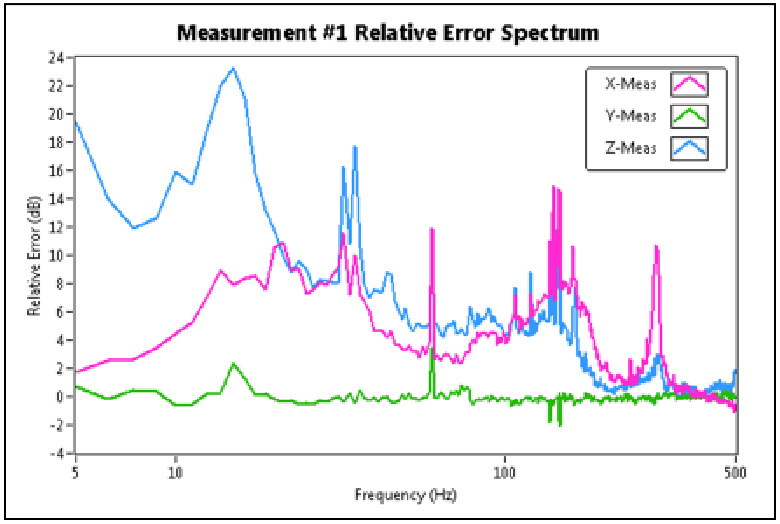




Global Error Applied to the Single Axis Test

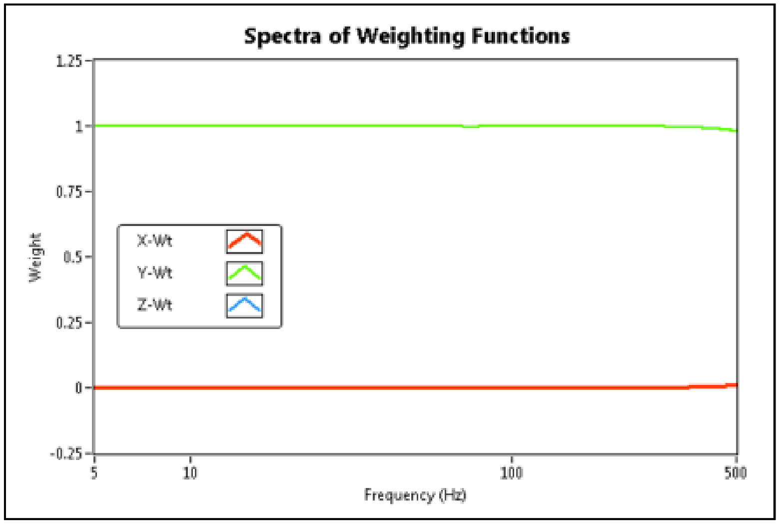
- Considering the Global Error of the Single Axis test highlights its usefulness
- Single Axis Y-axis (Transverse) test performed as a 3-DOF test
  - X & Z axes operated at null level
- Y axis excited per required profile
  - Meets +/-3dB relative error requirements
- Significant relative error on X & Z axes
  - Reference levels are set near or below system's noise floor – generates large error
- Error of X & Z should not dictate test acceptance
  - Not considered (or even measured) in a single axis configuration
- Global Error algorithm places the emphasis on Y-Axis error

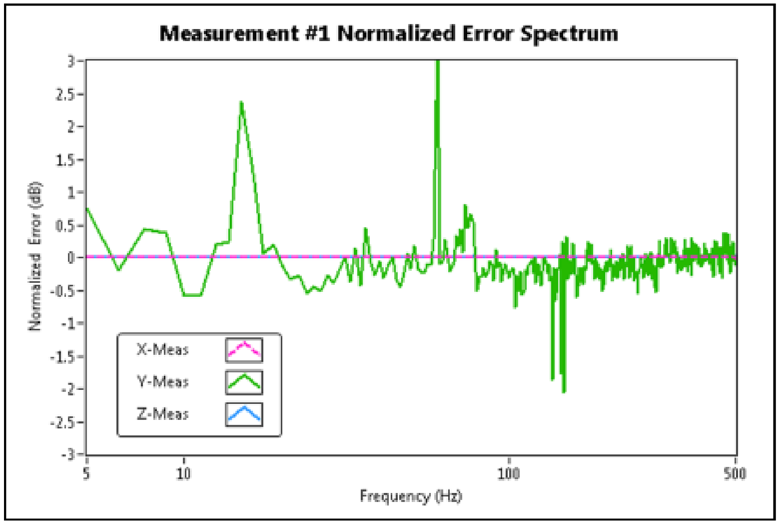


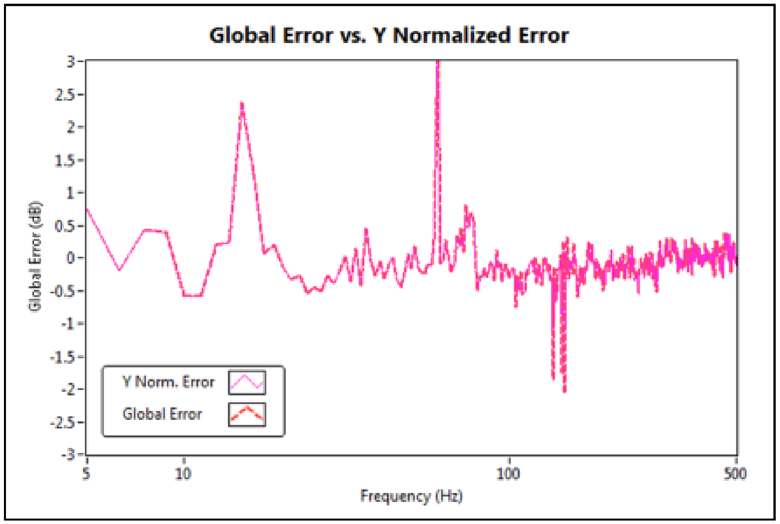


Single Axis Global Error Calculation

- Relative and Normalized error calculated for X, Y, & Z measurements
- Y-axis weighting becomes dominant
  - Nominally 1.0 over full bandwidth
  - X & Z weighting is null
- Global Error calculation accounts for the profile differences
  - Highlights the profile of interest
- Global error becomes a quantifiable acceptance criteria

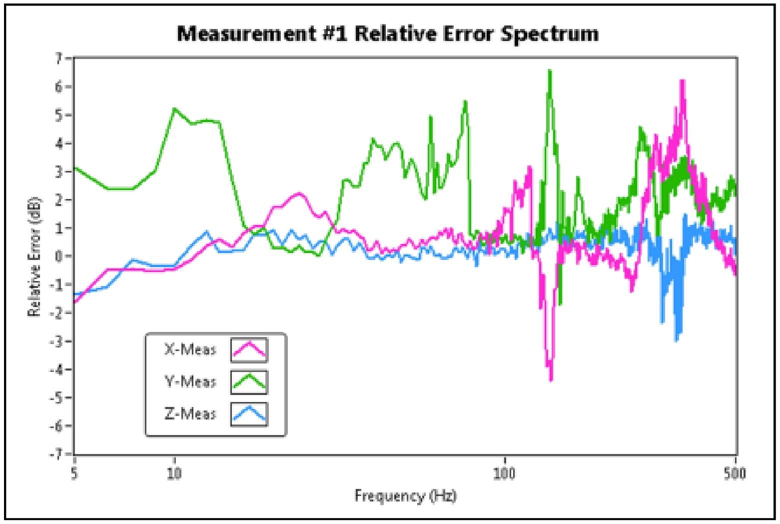


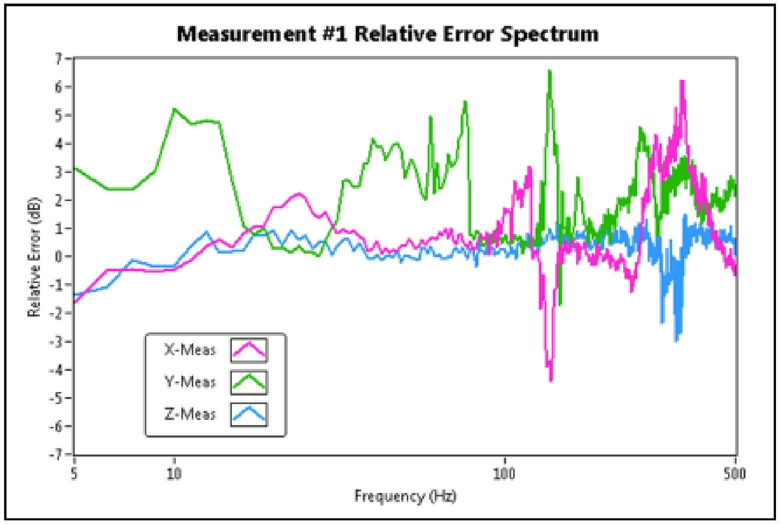




Conclusions

- A Global Error metric should be considered if a MIMO test has significant differences in DOF excitation levels
  - An order of magnitude or more constitutes a significant difference
  - Discuss and approve with appropriate test authority
- Provides more realistic approach to allowable errors
- Emphasizes the error of the dominant axes





- Research opportunities for examining how best to apply single axis standards to a MIMO test
- How are standard profiles applied to MIMO testing if the Cross Spectral Densities are unknown?

Questions / Comments

Acknowledgement: Dr. Michael Hale – Redstone Test Center, Dynamic Test Division